Technical Appendix to “The Equivalence of Wage and Price Staggering in Monetary Business Cycle Models”

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Abstract

This appendix details the derivation of a number of results reported in “The Equivalence of Wage and Price Staggering in Monetary Business Cycle Models,” which appears in the *Review of Economic Dynamics*.

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This appendix provides full derivations of the staggered wage and price models discussed in the *Review of Economic Dynamics* paper "The Equivalence of Wage and Price Staggering in Monetary Business Cycle Models."

The appendix is organized as follows. Section A presents the staggered wage model. Sections B and C derive the staggered price model with homogeneous and firm-specific factors, respectively. The final section compares the staggered price model with firm-specific factors to the corresponding firm-specific factor model derived by Chari, Kehoe, and McGratten (2000).

In the derivations that follow, I refer to a number of equations from the main text of the paper. These are denoted with a "p" to distinguish them from the appendix equations. For example, (5p) refers to equation (5) in the paper (describing the evolution of household i's holding of the capital stock), while (5) refers to appendix equation (5) (the demand for household i's differentiated labor).

A Derivation of the Staggered Wage Model

A.1 The Firm's Problem

The firm, taking as given the real wage on aggregate labor w_t and the real rental rate of capital r_t chooses aggregate labor h_t and capital k_t to minimize its cost of producing output y_t subject to its production function. Specifically, the firm solves:

$$\min_{\{h_t,k_t\}} w_t h_t + r_t k_t \text{ subject to } (h_t)^{1-\alpha} (k_t)^{\alpha} \geq y_t$$

where α represents the elasticity of output with respect to capital. The Lagrangian is written as:

$$\mathcal{L} = w_t h_t + r_t k_t - \lambda \left[(h_t)^{1-\alpha} (k_t)^{\alpha} - y_t \right].$$

The first-order conditions are:

$$w_t = \lambda (1-\alpha) \left(\frac{k_t}{h_t} \right)^{\alpha}, \; r_t = \lambda \alpha \left(\frac{h_t}{k_t} \right)^{1-\alpha}, \text{ and } y_t = (h_t)^{1-\alpha} (k_t)^{\alpha}.$$

The first two first-order conditions imply that $\frac{w_t}{r_t} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{k_t}{h_t}\right)$. Together with the third first-order condition, this gives the factor demand schedules:

$$h_t = \left(\frac{1-\alpha}{\alpha} \right)^{\alpha} Y_t \left(\frac{w_t}{r_t} \right)^{-\alpha} \text{ and } k_t = \left(\frac{\alpha}{1-\alpha} \right)^{1-\alpha} Y_t \left(\frac{w_t}{r_t} \right)^{1-\alpha}. \tag{1}$$

Substituting equations (1) into $w_t h_t + r_t k_t$ yields an expression for real total cost $tc_t = y_t \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t}{\alpha}\right)^{\alpha}$. Real marginal cost mc_t is therefore

$$mc_t = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t}{\alpha}\right)^{\alpha}. \tag{2}$$

A.2 The Intermediary's Problem

The intermediary, taking as given the real wages $\{w_t^i\}_{i=0}^1$ set by each of the i households for their differentiated labor input, chooses $\{h_t^i\}_{i=0}^1$ to minimize its production costs subject to the aggregator function. Specifically, the intermediary solves:

$$\min_{\{h_t^i\}_{i=0}^1} \int_0^1 w_t^i h_t^i di \text{ subject to } \left(\int_0^1 \left(h_t^i\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \geq h_t.$$

The Lagrangian is written as:

$$\mathcal{L} = \int_0^1 w_t^i h_t^i di - \lambda \left[\left(\int_0^1 \left(h_t^i\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} - h_t\right].$$

The first-order conditions are:

$$w_t^i = \lambda \left(\int_0^1 \left(h_t^i\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{1}{\sigma-1}} \left(h_t^i\right)^{-\frac{1}{\sigma}} \forall i \tag{3}$$

$$\left(\int_0^1 \left(h_t^i\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} = h_t. \tag{4}$$

If the left- and right-hand sides of equation (3) are both raised to the power $(1-\sigma)$:

$$\left(w_t^i\right)^{1-\sigma} = (\lambda)^{1-\sigma} \left(\int_0^1 \left(h_t^i\right)^{\frac{\sigma-1}{\sigma}} di\right)^{-1} \left(h_t^i\right)^{\frac{\sigma-1}{\sigma}}$$

and then integrated over the unit interval:

$$\int_0^1 \left(w_t^i\right)^{1-\sigma} di = (\lambda)^{1-\sigma} \left(\int_0^1 \left(h_t^i\right)^{\frac{\sigma-1}{\sigma}} di\right)^{-1} \left(\int_0^1 \left(h_t^i\right)^{\frac{\sigma-1}{\sigma}} di\right),$$

we are left with $\lambda = \left(\int_0^1 \left(w_t^i\right)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}$. This can be substituted for λ in equation (3) to yield

$$w_t^i = \left(\int_0^1 \left(w_t^i\right)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \underbrace{\left(\int_0^1 \left(h_t^i\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{1}{\sigma-1}}}_{\equiv (h_t)^{\frac{1}{\sigma}}} \left(h_t^i\right)^{-\frac{1}{\sigma}}.$$

Further manipulation yields the demand curves for each type of differentiated labor:

$$h_t^i = \left(\frac{w_t^i}{\left(\int_0^1 \left(w_t^i\right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}} \right)^{-\sigma} h_t. \tag{5}$$

To calculate the real cost of aggregate labor, note that the real total cost of producing h_t (call it tc_t^h) is equal to $\int_0^1 w_t^i h_t^i di$. Substituting in for each h_t^i using equation (5), we obtain that:

$$tc_t^h = h_t \left(\int_0^1 \left(w_t^i\right)^{1-\sigma} di \right)^{\frac{\sigma}{1-\sigma}} \int_0^1 \left(w_t^i\right)^{1-\sigma} di = h_t \left(\int_0^1 \left(w_t^i\right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}.$$

The firm sets the real wage on aggregate labor competitively, that is, equal to marginal cost mc_t^h, so that:

$$w_t = mc_t^h = \left(\int_0^1 \left(w_t^i\right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}. \tag{6}$$

This expression for the aggregate price of labor can be substituted into equation (5) to yield a simpler form for the firm's demand for household i's labor:

$$h_t^i = \left(\frac{w_t^i}{w_t} \right)^{-\sigma} h_t. \tag{7}$$

The assumption of two-period wage staggering implies that equation (6) can be written as:

$$w_t = \left(\int_0^{\frac{1}{2}} \left(\frac{X_t^w}{P_t} \right)^{1-\sigma} di + \int_{\frac{1}{2}}^1 \left(\frac{X_{t-1}^w}{P_t} \right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} = \left(\frac{1}{2} \left(\frac{X_t^w}{P_t} \right)^{1-\sigma} + \frac{1}{2} \left(\frac{X_{t-1}^w}{P_t} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{8}$$

where X_t^w is defined as the nominal wage that is set in period t.

A.3 The Household's Problem

A household i who is able to reset its nominal wage in period t takes as given the nominal interest rate, the gross inflation rate, the real rental rate on capital, the real wage rate on aggregate labor, aggregate labor demand, and N-period wage stickiness, and chooses its consumption c_t^i, real money balances $\frac{M_t^i}{P_t}$, nominal wage W_t^i, and capital stock k_t^i to maximize its utility (equation (3p)) subject to its budget constraint (equation (4p)), the evolution of the capital stock (equation (5p)), and the demand for its differentiated labor service ($h_t^i = h_t \cdot \left(\frac{W_t^i/P_t}{w_t}\right)^{-\sigma}$). Specifically, a household i who resets its nominal wage W_t^i in periods $\{Nk\}_{k=0}^{\infty}$ solves:

$$\max_{\left\{c_t^i, \frac{M_t^i}{P_t}, W_t^i, k_t^i\right\}_{t=0}^{\infty}} E_t \left[\sum_{t=0}^{\infty} \ln\left[\left(b\left(c_t^i\right)^v + (1-b)\left(\frac{M_t^i}{P_t}\right)^v\right)^{\frac{1}{v}}\right] + \eta \ln\left[1 - \left(\frac{W_t^i/P_t}{w_t}\right)^{-\sigma} h_t\right]\right]$$

$$\text{subject to } W_{Nk}^i = ... = W_{N(k+1)-1}^i \forall\ k \geq 0, \text{ and}$$

$$B_t^i + M_t^i \leq R_{t-1}B_{t-1}^i + M_{t-1}^i + P_t \frac{\left(W_t^i/P_t\right)^{1-\sigma}}{\left(w_t\right)^{\sigma}} h_t + P_t r_t k_t - P_t c_t^i - P_t k_t^i J^{-1}\left(\frac{k_{t+1}^i}{k_t^i} - 1 + \delta\right). \quad (9)$$

A household i who is unable able to reset its nominal wage in period t solves a similar problem but takes its preset wage W_t^i as given.

The first-order conditions for real money balances, consumption, and the capital stock for all households are given by:

$$\left(1 - R_t^{-1}\right)\frac{\beta^t}{c_t^i}\frac{b\left(c_t^i\right)^v}{b\left(c_t^i\right)^v + (1-b)\left(\frac{M_t^i}{P_t}\right)^v} = \frac{\beta^t}{\frac{M_t^i}{P_t}}\frac{(1-b)\left(\frac{M_t^i}{P_t}\right)^v}{b\left(c_t^i\right)^v + (1-b)\left(\frac{M_t^i}{P_t}\right)^v}, \quad (10)$$

$$\frac{\beta^t}{P_t c_t^i}\frac{b\left(c_t^i\right)^v}{b\left(c_t^i\right)^v + (1-b)\left(\frac{M_t^i}{P_t}\right)^v} = E_t\left[\frac{R_t\beta^{t+1}}{P_{t+1}c_{t+1}^i}\frac{b\left(c_{t+1}^i\right)^v}{b\left(c_{t+1}^i\right)^v + (1-b)\left(\frac{M_{t+1}^i}{P_{t+1}}\right)^v}\right] = 0, \text{ and} \quad (11)$$

$$\frac{\beta^t}{c_t^i}\frac{b\left(c_t^i\right)^v}{b(c_t^i)^v + (1-b)\left(\frac{M_t^i}{P_t}\right)^v}\left(J^{-1'}\left(\frac{k_{t+1}^i}{k_t^i} - 1 + \delta\right)\right) \quad (12)$$

$$= E_t\left[\frac{\beta^{t+1}}{c_{t+1}^i}\frac{b\left(c_{t+1}^i\right)^v}{b\left(c_{t+1}^i\right)^v + (1-b)\left(\frac{M_{t+1}^i}{P_{t+1}}\right)^v}\left(r_{t+1} - J^{-1}\left(\frac{k_{t+2}^i}{k_{t+1}^i} - 1 + \delta\right) - \frac{k_{t+2}^i}{k_{t+1}^i}J^{-1'}\left(\frac{k_{t+2}^i}{k_{t+1}^i} - 1 + \delta\right)\right)\right].$$

Equation (10) simplifies to:

$$\frac{M_t}{P_t} = c_t\left(\left(\frac{b}{1-b}\right)\left(1 - \frac{1}{R_t}\right)\right)^{-\frac{1}{1-v}}. \quad (13)$$

Equations (11) and (12) also simplify and equation (13) can be substituted for $\frac{M_t^i}{P_t}$ to yield:

$$U_c\left(c_t, R_t\right) = E_t\left[\frac{R_t\beta}{\Pi_{t+1}}U_c\left(c_{t+1}, R_{t+1}\right)\right] \text{ and} \quad (14)$$

$$
\begin{aligned}
& U_c\left(c_t, R_t\right)\left(J^{-1'}\left(\frac{k_{t+1}}{k_t}-1+\delta\right)\right) && (15)\\
= \quad & E_t\left[\beta U_c\left(c_{t+1}, R_{t+1}\right)\left(r_{t+1}-J^{-1}\left(\frac{k_{t+2}}{k_{t+1}}-1+\delta\right)-\frac{k_{t+2}}{k_{t+1}} J^{-1'}\left(\frac{k_{t+2}}{k_{t+1}}-1+\delta\right)\right)\right]
\end{aligned}
$$

where $U_c\left(c_t, R_t\right)=\frac{1}{c_t} \cdot \frac{b^{\frac{1}{1-v}}}{b^{\frac{1}{1-v}}+(1-b)^{\frac{1}{1-v}}\left(1-\frac{1}{R_t}\right)^{-\frac{v}{1-v}}}$. The first-order condition for wages for the households able to set their wages in period t (assuming that N is two periods) is:

$$
\begin{aligned}
0 \quad = \quad & \beta^t\left[U_c\left(c_t, R_t\right)\left(\frac{X_t^w / P_t}{w_t}\right)^{-\sigma} \frac{h_t(1-\sigma)}{P_t}+\frac{\sigma \eta\left(\frac{X_t^w / P_t}{w_t}\right)^{-\sigma} h_t}{1-\left(\frac{X_t^w / P_t}{w_t}\right)^{-\sigma} h_t} \frac{1}{X_t^w / P_t}\right] && (16)\\
& +\beta^{t+1} E_t\left[U_c\left(c_{t+1}, R_{t+1}\right)\left(\frac{X_t^w / P_t}{\Pi_{t+1} w_{t+1}}\right)^{-\sigma} \frac{h_{t+1}(1-\sigma)}{P_{t+1}}+\frac{\sigma \eta\left(\frac{X_t^w / P_t}{\Pi_{t+1} w_{t+1}}\right)^{-\sigma} h_{t+1}}{1-\left(\frac{X_t^w / P_t}{\Pi_{t+1} w_{t+1}}\right)^{-\sigma} h_{t+1}} \frac{1}{X_t^w / P_t}\right] .
\end{aligned}
$$

Equation (16) simplifies to:

$$
\frac{X_t^w}{P_t}=\frac{\sigma}{\sigma-1} \cdot \frac{h_t \frac{\eta}{1-\left(\frac{X_t^w / P_t}{w_t}\right)^{-\sigma} h_t}+\beta E_t\left[h_{t+1}\left(\frac{w_t}{\Pi_{t+1} w_{t+1}}\right)^{-\sigma} \frac{\eta}{1-\left(\frac{X_t^w / P_t}{\Pi_{t+1} w_{t+1}}\right)^{-\sigma} h_{t+1}}\right]}{h_t U_c\left(c_t, R_t\right)+\beta E_t\left[h_{t+1}\left(\frac{1}{\Pi_{t+1}}\right)\left(\frac{w_t}{\Pi_{t+1} w_{t+1}}\right)^{-\sigma} U_c\left(c_{t+1}, R_{t+1}\right)\right]} . \qquad (17)
$$

Note that in writing household i's first-order conditions above I have dropped the i superscript from c_t, M_t, and k_t; the implication is that the values of these variables are the same across all households. In general this would not be the case since households receive different wages and work different hours depending on whether they are members of $\left[0, \frac{1}{2}\right]$ or $\left(\frac{1}{2}, 1\right]$; as a result, their accumulated wealth and thus their c_t, M_t, and k_t profiles are likely to differ. To allow a single c_t, M_t, and k_t profile to characterize all households requires the assumption that asset portfolios can be constructed so as to provide the household with complete insurance against any idiosyncratic risk. Consequently, a household's wealth is independent of the period in which it sets its wage. Since $c_t=\int_0^1 c_t^i di$, $M_t=\int_0^1 M_t^i di$, and $k_t=\int_0^1 k_t^i di$, this assumption allows the i superscripts to be dropped from consumption, real money balances, and the capital stock in equations (13) to (15). The i superscripts remain on h_t^i and W_t^i since wages and hours worked will vary by household depending on the period in which the firm resets its nominal wage; the variable h_t^i, however, does not appear in equations (16) and (17) since it has been substituted out with equation (7) and the variable W_t^i in equations (16) and (17) appears only for firms resetting wages in period t and has been replaced with the variable X_t^w.

A.4 Solving the Fully Specified Model

Equilibrium in this economy consists of an allocation $\{\{h_t^i\}_{i=0}^{1}, h_t, k_t, c_t, \frac{M_t}{P_t}, y_t\}_{t=0}^{\infty}$ and sequence $\{\Pi_t, \frac{X_t^w}{P_t}, w_t, r_t, \mu_t, R_t, mc_t\}_{t=0}^{\infty}$. The equilibrium allocation and sequence satisfy the following conditions: (i) the first-order conditions from the firm's cost-minimization problem (1p) (equations (1) and (2)); (ii) the first-order conditions from the intermediary's cost-minimization problem (2p) (equations (7) and (8)); (iii) the first-order conditions from the households' utility-maximization problems (6p) and (7p) (equations (13) to (15) and (17)); (iv) the monetary authority follows (8p); (v) the goods market clears ($y_t = c_t + k_t J^{-1}\left(\frac{k_{t+1}}{k_t} - 1 + \delta\right)$); and (vi) factor markets clear. This is given the initial conditions, k_0, μ_{-1}, $\frac{M_{-1}}{P_{-1}}$, $\frac{X_{-1}^w}{P_{-1}}$, and the sequence of monetary policy shocks $\{\varepsilon_t\}_{t=0}^{\infty}$. The model's log-linearized equilibrium conditions are given in table A.1.

Table A.1

$\widehat{\mu}_t = \zeta\widehat{\mu}_{t-1} + \varepsilon_t$	Eq. (8p)
$\widehat{h}_t = \widehat{y}_t - \alpha\widehat{w}_t + \alpha\widehat{r}_t$	Eq. (1)
$\widehat{k}_t = \widehat{y}_t + (1-\alpha)\,\widehat{w}_t - (1-\alpha)\,\widehat{r}_t$	Eq. (1)
$\widehat{mc}_t = (1-\alpha)\,\widehat{w}_t + \alpha\widehat{r}_t = 0$	Eq. (2)
$\widehat{h}_t^i = \widehat{h}_t - \sigma\widehat{\frac{X_t^w}{P_t}} + \sigma\widehat{w}_t$ for $i \in \left[0, \frac{1}{2}\right]$	Eq. (7)
$\widehat{h}_t^i = \widehat{h}_t - \sigma\widehat{\frac{X_{t-1}^w}{P_{t-1}}} + \sigma\widehat{w}_t + \sigma\widehat{\Pi}_t$ for $i \in \left(\frac{1}{2}, 1\right]$	Eq. (7)
$2\widehat{w}_t = \widehat{\frac{X_t^w}{P_t}} + \widehat{\frac{X_{t-1}^w}{P_{t-1}}} - \widehat{\Pi}_t$	Eq. (8, 13p)
$\widehat{\frac{M_t}{P_t}} = \widehat{c}_t - \left(\frac{1}{1-v}\right)\left(\frac{\frac{1}{R^*}}{1-\frac{1}{R^*}}\right)\widehat{R}_t$	Eq. (13)
$-\rho_{cc}E_t\widehat{c}_{t+1} - \rho_{cr}E_t\widehat{R}_{t+1} = -\rho_{cc}\widehat{c}_t + (1-\rho_{cr})\,\widehat{R}_t - E_t\widehat{\Pi}_{t+1}$	Eq. (14)
$\frac{1}{1+\delta\beta-\beta}\left(\widehat{R}_t - E_t\widehat{\Pi}_{t+1}\right) = E_t\widehat{r}_{t+1} + \frac{J^{-1''}(\delta)}{1+\delta\beta-\beta}\left(\beta E_t\widehat{k}_{t+2} - (1+\beta)\,\widehat{k}_{t+1} + \widehat{k}_t\right)$	Eq. (15)
$\widehat{\widetilde{\omega}}_t = \frac{1}{1+\beta}\left(\rho_{hh}\widehat{h}_t^i + \beta\rho_{hh}\widehat{h}_{t+1}^i - \rho_{cc}\widehat{c}_t - \beta\rho_{cc}\widehat{c}_{t+1} - \rho_{cr}\widehat{R}_t - \beta\rho_{cr}\widehat{R}_{t+1}\right)$	Eq. (17)
$\widehat{y}_t = \frac{c^*}{y^*}\widehat{c}_t + \left(1 - \frac{c^*}{y^*}\right)\left(\frac{1}{\delta}E_t\widehat{k}_{t+1} - \frac{1-\delta}{\delta}\widehat{k}_t\right)$	Y-Clearing

Here, $\rho_{cc} = -1$, $\rho_{cr} = \frac{-(1-b)^{\frac{1}{1-v}}\left(1-\frac{1}{R^*}\right)^{\frac{-v}{1-v}}}{b^{\frac{1}{1-v}}+(1-b)^{\frac{1}{1-v}}\left(1-\frac{1}{R^*}\right)^{\frac{-v}{1-v}}}\frac{\frac{1}{R^*}}{1-\frac{1}{R^*}}\frac{-v}{1-v}$, $\rho_{hh} = \frac{h^*}{1-h^*}$, and $\frac{c^*}{y^*} = 1 - \frac{\delta\alpha}{\widetilde{\rho}^*}$ (where $\widetilde{\rho}^*$, the equilibrium real rental rate, is equal to $\frac{1}{\beta} - 1 + \delta$). In general, I calibrate the model with the parameter values used by Huang and Liu (1999); these are summarized and discussed in section 1.7.1 of my paper. The log-linearized first-order conditions given in table A.1 can be reduced to the system of difference equations described by equation (9p) in section 1.7.1.

A.5 Solving the Simplified Model

Equilibrium in the core model of section 1.7.2 of the paper is an allocation $\{y_t\}_{t=0}^{\infty}$ and a sequence $\{\frac{X_t^w}{P_t}, R_t\}_{t=0}^{\infty}$ that satisfy equations (10p) to (12p), with the equilibrium conditions noted in points (a) to (e) of section 1.7.2 imposed. Specifically,

$$\frac{1}{y_t}\cdot\frac{1}{2-\frac{1}{R_t}} = R_t\beta E_t\left[\frac{1}{y_{t+1}}\cdot\frac{1}{2-\frac{1}{R_{t+1}}}\cdot\frac{\left(2-\left(X_{t+1}^w/P_{t+1}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}{X_t^w/P_t}\right], \tag{18}$$

$$\frac{X_t^w}{P_t} = \frac{\sigma}{\sigma-1}\cdot\frac{\frac{\eta y_t}{1-y_t\left(X_t^w/P_t\right)^{-\sigma}} + \beta E_t\left[\frac{\left(X_t^w/P_t\right)^{\sigma}}{\left(2-\left(X_{t+1}^w/P_{t+1}\right)^{1-\sigma}\right)^{\frac{\sigma}{1-\sigma}}}\cdot\frac{\eta y_{t+1}}{1-y_{t+1}\left(2-\left(X_{t+1}^w/P_{t+1}\right)^{1-\sigma}\right)^{\frac{-\sigma}{1-\sigma}}}\right]}{\frac{1}{2-\frac{1}{R_t}} + \beta E_t\left[\frac{2-\left(X_{t+1}^w/P_{t+1}\right)^{1-\sigma}}{\left(X_t^w/P_t\right)^{1-\sigma}}\cdot\frac{1}{2-\frac{1}{R_{t+1}}}\right]}, \tag{19}$$

$$\text{and}\quad \frac{\left(2-\left(X_t^w/P_t\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}{X_{t-1}^w/P_{t-1}}y_t = y_{t-1}\mu^*\exp\left[\varepsilon_t\right]. \tag{20}$$

This is given y_{-1} and $\frac{X_{-1}^w}{P_{-1}}$ and the sequence of monetary policy shocks $\{\varepsilon_t\}_{t=0}^{\infty}$.

Equations (18) to (20) can be log-linearized to yield equations (14p) to (16p) in section 1.7.2 of the paper. Of the three equations that characterize equilibrium in the simplified staggered-wage model only equation (19) is somewhat arduous to log-linearize. This equation log-linearizes as follows:

$$\begin{aligned}
\widehat{X}_t^w - \widehat{P}_t \;=\;& \frac{1}{1+\beta}\left(\left(1+\rho_{hh}\right)\widehat{y}_t - \sigma\rho_{hh}\left(\widehat{X}_t^w - \widehat{P}_t\right)\right) + \frac{1}{1+\beta}\left(\frac{\frac{1}{R^*}}{2-\frac{1}{R^*}}\widehat{R}_t\right)\\
&+\frac{\beta}{1+\beta}\left(\left(1+\rho_{hh}\right)E_t\widehat{y}_{t+1} + \sigma\left(\widehat{X}_t^w - \widehat{P}_t\right) + \sigma\left(1+\rho_{hh}\right)\left(E_t\widehat{X}_{t+1}^w - E_t\widehat{P}_{t+1}\right)\right)\\
&+\frac{\beta}{1+\beta}\left(\frac{\frac{1}{R^*}}{2-\frac{1}{R^*}}E_t\widehat{R}_{t+1} + \left(1-\sigma\right)\left(\widehat{X}_t^w - \widehat{P}_t\right) + \left(1-\sigma\right)\left(E_t\widehat{X}_{t+1}^w - E_t\widehat{P}_{t+1}\right)\right)
\end{aligned}$$

where ρ_{hh}, the elasticity of labor substitution, is: $\frac{V''(h^*)h^*}{V'(h^*)} = \frac{h^*}{1-h^*}$. This rearranges to:

$$\begin{aligned}
\left(1+\sigma\rho_{hh}\right)\left(\widehat{X}_t^w - \widehat{P}_t\right) \;=\;& \left(1+\rho_{hh}\right)\left(\widehat{y}_t + \beta E_t\widehat{y}_{t+1}\right) + \frac{\frac{1}{R^*}}{2-\frac{1}{R^*}}\left(\widehat{R}_t + \beta E_t\widehat{R}_{t+1}\right)\\
&+\beta\left(1+\sigma\rho_{hh}\right)\left(E_t\widehat{X}_{t+1}^w - E_t\widehat{P}_{t+1}\right).
\end{aligned}$$

Dividing through on both sides by $(1+\sigma\rho_{hh})$ and setting β, the discount factor (and by implication, R^*, the gross nominal interest rate) equal to unity yields equation (15p).

The equilibrium paths of $\widehat{y}_t$, $\widehat{\frac{X_t^w}{P_t}}$, and $\widehat{R}_t$ can be found from the log-linearized system (equations (14p) to (16p)). The equilibrium path of $\widehat{R}_t$ can be derived immediately. By taking equation (16p) forward one period and then taking expectations for period t one finds that the left-hand side of equation (14p) is equal to zero. This means that $E_t\widehat{R}_{t+1} = 2\widehat{R}_t$, which implies that $\widehat{R}_t = \frac{1}{2}E_t\widehat{R}_{t+1} = \lim_{k\to\infty}(\frac{1}{2})^k E_t\widehat{R}_{t+k} = 0$. This finding eliminates $\widehat{R}_t$ and $E_t\widehat{R}_{t+1}$ from the log-linearized labor supply schedule (equation (15p)), so yielding:

$$\widehat{X}_t^w - \widehat{P}_t = \gamma\left(\widehat{y}_t + E_t\widehat{y}_{t+1}\right) + \left(E_t\widehat{X}_{t+1}^w - E_t\widehat{P}_{t+1}\right). \tag{21}$$

The log-linearized expressions for money demand and the market-clearing condition ($\widehat{M}_t - \widehat{P}_t = \widehat{c}_t = \widehat{y}_t$) can be substituted for $\widehat{y}_t$ in equation (21) to yield:

$$\widehat{X}_t^w - \widehat{P}_t = \gamma\left(\widehat{M}_t - \widehat{P}_t + E_t\widehat{M}_{t+1} - E_t\widehat{P}_{t+1}\right) + \left(E_t\widehat{X}_{t+1}^w - E_t\widehat{P}_{t+1}\right). \tag{22}$$

The price level can be eliminated from equation (22) by noting that equation (13p) log-linearizes to $\widehat{P}_t = \frac{1}{2}\widehat{X}_t^w + \frac{1}{2}\widehat{X}_{t-1}^w$; substituting this into equation (22) yields a second-order difference equation in $\widehat{X}^w$ with $\widehat{M}$ as the driving process:

$$E_t\widehat{X}_{t+1}^w - 2\left(\frac{1+\gamma}{1-\gamma}\right)\widehat{X}_t^w + \widehat{X}_{t-1}^w = -\frac{2\gamma}{1-\gamma}\left(\widehat{M}_t + E_t\widehat{M}_{t+1}\right).$$

The variables $E_t\widehat{X}_{t+1}^w$, $\widehat{X}_t^w$, and $\widehat{X}_{t-1}^w$ can be expressed using lag operators and the symmetric lag polynomial can be factorized to obtain:

$$L^{-1}\left(L^2 - 2\left(\frac{1+\gamma}{1-\gamma}\right)L + 1\right)\widehat{X}_t^w = L^{-1}\left(a - L\right)\left(a^{-1} - L\right)\widehat{X}_t^w = -\frac{2\gamma}{1-\gamma}\left(\widehat{M}_t + E_t\widehat{M}_{t+1}\right) \tag{23}$$

where $a = \frac{1-\sqrt{\gamma}}{1+\sqrt{\gamma}}$. Note that since $\gamma > 0$, $|a| < 1$. Equation (23) can be re-written as:

$$a^{-1}\left(1 - aL^{-1}\right)\left(aL - 1\right)\widehat{X}_t^w = -\frac{2\gamma}{1-\gamma}\left(\widehat{M}_t + E_t\widehat{M}_{t+1}\right)$$

and re-arranged to

$$\left(aL - 1\right)\widehat{X}_t^w = -\frac{2\gamma a}{1-\gamma}\cdot\frac{1}{1-aL^{-1}}\left(\widehat{M}_t + E_t\widehat{M}_{t+1}\right) = -\frac{2\gamma a}{1-\gamma}\sum_{s=0}^{\infty}(a)^s E_t\left(\widehat{M}_{t+s} + E_t\widehat{M}_{t+1+s}\right),$$

or alternatively

$$\widehat{X}_t^w = a\widehat{X}_{t-1}^w + \frac{2\gamma a}{1-\gamma}\sum_{s=0}^{\infty}(a)^s E_t\left(\widehat{M}_{t+s} + E_t\widehat{M}_{t+1+s}\right). \tag{24}$$

The money supply process given by (12p) (which log-linearizes to $\widehat{M}_t = \widehat{M}_{t-1} + \varepsilon_t$) implies that $E_t \widehat{M}_{t+s} = \widehat{M}_t$ for all values of $s > 0$. Equation (24) thus becomes:

$$\widehat{X}_t^w = a\widehat{X}_{t-1}^w + \frac{2\gamma a}{1-\gamma} \sum_{s=0}^{\infty} (a)^s \, 2\widehat{M}_t = a\widehat{X}_{t-1}^w + \frac{2\sqrt{\gamma}}{1+\sqrt{\gamma}} \widehat{M}_t = a\widehat{X}_{t-1}^w + (1-a)\,\widehat{M}_t. \tag{25}$$

Equation (25) can be substituted into the log-linearized version of equation (13p) to give:

$$\begin{aligned} \widehat{P}_t &= \frac{1}{2}\widehat{X}_t^w + \frac{1}{2}\widehat{X}_{t-1}^w = \frac{1}{2}\left(a\widehat{X}_{t-1}^w + (1-a)\,\widehat{M}_t\right) + \frac{1}{2}\left(a\widehat{X}_{t-2}^w + (1-a)\,\widehat{M}_{t-1}\right) \\ &= a\underbrace{\left(\frac{1}{2}\widehat{X}_{t-1}^w + \frac{1}{2}\widehat{X}_{t-2}^w\right)}_{\equiv P_{t-1}} + \frac{1}{2}(1-a)\left(\widehat{M}_t + \widehat{M}_{t-1}\right) = a\widehat{P}_{t-1} + \frac{1}{2}(1-a)\left(\widehat{M}_t + \widehat{M}_{t-1}\right). \end{aligned} \tag{26}$$

I substitute for $\widehat{P}_t$ using $\widehat{P}_t = \widehat{M}_t - \widehat{y}_t$ which yields:

$$\widehat{M}_t - \widehat{y}_t = a\left(\widehat{M}_{t-1} - \widehat{y}_{t-1}\right) + \frac{1}{2}(1-a)\left(\widehat{M}_t + \widehat{M}_{t-1}\right). \tag{27}$$

Re-arranging equation (27) yields the equilibrium path of $\widehat{y}_t$ given $\widehat{y}_{-1}$ and the sequence of shocks $\{\varepsilon_t\}_{t=0}^{\infty}$:

$$\widehat{y}_t = a\widehat{y}_{t-1} + \frac{1}{2}(1+a)\left(\widehat{M}_t - \widehat{M}_{t-1}\right) = a\widehat{y}_{t-1} + \frac{1}{1+\sqrt{\gamma}}\varepsilon_t. \tag{28}$$

The equilibrium path of $\{\widehat{\frac{X_t^w}{P_t}}\}_{t=0}^{\infty}$ can be found by re-writing equation (25) and the log-linearized versions of equation (13p) ($\widehat{P}_t = \frac{1}{2}\widehat{X}_t^w + \frac{1}{2}\widehat{X}_{t-1}^w$) as:

$$\widehat{X}_t^w - \widehat{P}_t = a\left(\widehat{X}_{t-1}^w - \widehat{P}_{t-1}\right) - a\left(\widehat{P}_t - \widehat{P}_{t-1}\right) + (1-a)\left(\widehat{M}_t - \widehat{P}_t\right) \text{ and} \tag{29}$$

$$\widehat{P}_t - \widehat{P}_{t-1} = \left(\widehat{X}_t^w - \widehat{P}_t\right) + \left(\widehat{X}_{t-1}^w - \widehat{P}_{t-1}\right). \tag{30}$$

Equation (30) can be substituted for $(\widehat{P}_t - \widehat{P}_{t-1})$ in equation (29) while $\widehat{y}_t$ can be substituted for $(\widehat{M}_t - \widehat{P}_t)$. This yields

$$(1+a)\left(\widehat{X}_t^w - \widehat{P}_t\right) = (1-a)\,\widehat{y}_t,$$

which implies that the equilibrium path of $\widehat{\frac{X_t^w}{P_t}}$ given $\widehat{y}_{-1}$ and the sequence of shocks $\{\varepsilon_t\}_{t=0}^{\infty}$ is

$$\widehat{X}_t^w - \widehat{P}_t = \sqrt{\gamma}\widehat{y}_t = a\sqrt{\gamma}\widehat{y}_{t-1} + \frac{\sqrt{\gamma}}{1+\sqrt{\gamma}}\varepsilon_t.$$

Thus the responses of $\widehat{y}_t$, $\widehat{\frac{X_t^w}{P_t}}$, and $\widehat{R}_t$, to a monetary shock ε_0 (given that $y_{-1} = 0$) can be written as equation (25p).

B Derivation of the Staggered Price Model

B.1 The Firm's Problem

The first part of firm j's problem is to choose labor h_t^j and capital k_t^j to minimize production costs, taking as given the real wage on homogeneous labor, w_t, the real rental rate of homogeneous capital, r_t, and the production function. Specifically, firm j solves:

$$\min_{\{h_t^j, k_t^j\}} w_t h_t^j + r_t k_t^j \text{ subject to } \left(h_t^j\right)^{1-\alpha} \left(k_t^j\right)^{\alpha} \geq y_t^j.$$

This problem is very similar to that solved by the firm in the staggered wage model detailed in section A.1. The only difference is that in section A.1 the variable w_t denoted the real wage on the aggregate labor stock used in the firms' production process, while now the variable w_t denotes the real wage on the homogeneous labor stock that the firm uses. The steps taken to solve the problem are exactly the same as those followed in section A.1, and the solutions that emerge are similar in form, that is:

$$h_t^j = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} y_t^j \left(\frac{w_t}{r_t}\right)^{-\alpha}, \; k_t^j = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} y_t^j \left(\frac{w_t}{r_t}\right)^{1-\alpha}, \text{ and } mc = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t}{\alpha}\right)^{\alpha}. \tag{31}$$

The problem for firms who set new prices in periods $\{Nk\}_{k=0}^{\infty}$ is to choose $\{P_t^j\}_{t=0}^{\infty}$ so as to maximize the present discounted value of their profits, taking as given the real marginal cost of producing y_t^j, the aggregate price level, aggregate demand, the nominal interest rate, N-period price stickiness, and the demand curve it faces for y_t^j. Specifically, the firm solves:

$$\max_{\{P_t^j\}_{t=0}^{\infty}} E_t \left\{ \sum_{t=0}^{\infty} Q_{0,t} \left(P_t^j - P_t mc_t^j\right) y_t^j \right\} \text{ where } Q_{0,0} = 1 \text{ and } Q_{0,t} = \prod_{s=0}^{t-1} \frac{1}{R_s} t \geq 1$$

$$\text{subject to } y_t^j = y_t \left(\frac{P_t^j}{P_t}\right)^{-\theta} \text{ and } P_{Nk}^i = ... = P_{N(k+1)-1}^i \forall k \geq 0.$$

In choosing the price P_t^j that will remain in effect for the next N periods (where N here is assumed to equal two) the firm solves:

$$\max_{P_t^j} \left(P_t^j - P_t mc_t^j\right) y_t \left(\frac{P_t^j}{P_t}\right)^{-\theta} + \frac{1}{R_t} E_t \left[\left(P_t^j - P_{t+1} mc_{t+1}^j\right) y_{t+1} \left(\frac{P_t^j}{P_{t+1}}\right)^{-\theta} \right]. \tag{32}$$

The first-order condition is:

$$0 = (1-\theta) \left(\frac{P_t^j}{P_t}\right)^{-\theta} y_t + \theta \left(\frac{P_t^j}{P_t}\right)^{-\theta} \frac{P_t mc_t^j y_t}{P_t^j}$$

$$+\frac{1}{R_t}E_t\left[(1-\theta)\left(\frac{P_t^j}{P_{t+1}}\right)^{-\theta}y_{t+1}+\theta\left(\frac{P_t^j}{P_{t+1}}\right)^{-\theta}\frac{P_{t+1}mc_{t+1}^j y_{t+1}}{P_t^j}\right]$$

which can be rewritten as:

$$\begin{aligned}0 &= (1-\theta)\left(\frac{X_t^p}{P_t}\right)^{-\theta}y_t+\theta\left(\frac{X_t^p}{P_t}\right)^{-\theta-1}mc_t^j y_t\\ &\quad+\frac{1}{R_t}E_t\left[(1-\theta)\left(\frac{X_t^p}{P_t}\cdot\frac{1}{\Pi_{t+1}}\right)^{-\theta}y_{t+1}+\theta\left(\frac{X_t^p}{P_t}\cdot\frac{1}{\Pi_{t+1}}\right)^{-\theta-1}mc_{t+1}^j y_{t+1}\right]\end{aligned}$$

where $\frac{X_t^p}{P_t}$ denotes the ratio of prices set this period (X_t^p) to the aggregate price level (P_t) and mc_t^j denotes the real marginal cost of production for firm j in period t. Dividing through by $\left(\frac{X_t^p}{P_t}\right)^{-\theta-1}$ yields:

$$(1-\theta)\frac{X_t^p}{P_t}y_t+\theta mc_t^j y_t+\frac{1}{R_t}E_t\left[(1-\theta)\left(\frac{1}{\Pi_{t+1}}\right)^{-\theta}\frac{X_t^p}{P_t}y_{t+1}+\theta\left(\frac{1}{\Pi_{t+1}}\right)^{-\theta-1}mc_{t+1}^j y_{t+1}\right]=0,$$

which when rearranged yields the first-order condition for prices for firm j where $j\in\left[0,\frac{1}{2}\right]$:

$$\frac{X_t^p}{P_t}=\left(\frac{\theta}{\theta-1}\right)\frac{y_t mc_t+E_t\left[\left(\frac{\Pi_{t+1}}{R_t}\right)y_{t+1}mc_{t+1}\left(\frac{1}{\Pi_{t+1}}\right)^{-\theta}\right]}{y_t+\left(\frac{1}{R_t}\right)E_t\left[y_{t+1}\left(\frac{1}{\Pi_{t+1}}\right)^{-\theta}\right]}. \tag{33}$$

This is equation (31p) in section 2.7.2 of the paper. Note that since the real wage and rental rate are the same across all firms, real marginal cost is also the same, so mc_t can be written without the j superscript.

B.2 The Intermediary's Problem

The intermediary takes as given the prices $\{P_t^j\}_{i=0}^1$ set by each firm for its differentiated output, and chooses $\{y_t^j\}_{i=0}^1$ to minimize its production costs subject to the aggregator function. Specifically, the intermediary solves:

$$\max_{\{y_t^j\}_{j=0}^1}\int_0^1 P_t^j y_t^j dj \text{ subject to }\left(\int_0^1\left(y_t^j\right)^{\frac{\theta-1}{\theta}}dj\right)^{\frac{\theta}{\theta-1}}\geq y_t.$$

The Lagrangian is written as:

$$\mathcal{L}=\int_0^1 P_t^j y_t^j dj-\lambda\left[\left(\int_0^1\left(y_t^j\right)^{\frac{\theta-1}{\theta}}dj\right)^{\frac{\theta}{\theta-1}}-y_t\right].$$

The first-order conditions are:

$$P_t^j = \lambda \left(\int_0^1 \left(y_t^j \right)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{1}{\theta-1}} \left(y_t^j \right)^{-\frac{1}{\theta}} \forall i \tag{34}$$

$$\left(\int_0^1 \left(y_t^j \right)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} = y_t. \tag{35}$$

If the left- and right-hand sides of equation (34) are both raised to the power $(1-\theta)$:

$$\left(P_t^j \right)^{1-\theta} = (\lambda)^{1-\theta} \left(\int_0^1 \left(y_t^j \right)^{\frac{\theta-1}{\theta}} dj \right)^{-1} \left(y_t^j \right)^{\frac{\theta-1}{\theta}}$$

and then integrated over the unit interval:

$$\int_0^1 \left(P_t^j \right)^{1-\theta} dj = (\lambda)^{1-\theta} \left(\int_0^1 \left(y_t^j \right)^{\frac{\theta-1}{\theta}} dj \right)^{-1} \left(\int_0^1 \left(y_t^j \right)^{\frac{\theta-1}{\theta}} dj \right)$$

we are left with $\lambda = \left(\int_0^1 \left(P_t^j \right)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}$. This can be substituted for λ in equation (34) to yield:

$$P_t^j = \left(\int_0^1 \left(P_t^j \right)^{1-\theta} dj \right)^{\frac{1}{1-\theta}} \underbrace{\left(\int_0^1 \left(y_t^j \right)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{1}{\theta-1}}}_{\equiv (y_t)^{\frac{1}{\theta}}} \left(y_t^j \right)^{-\frac{1}{\theta}}.$$

Further manipulation yields the demand curves for each type of differentiated output:

$$y_t^j = \left(\frac{P_t^j}{\left(\int_0^1 \left(P_t^j \right)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}} \right)^{-\theta} y_t. \tag{36}$$

To calculate the price of aggregate output (P_t) one notes that the nominal total cost in period t of producing y_t is equal to $\int_0^1 P_t^j y_t^j dj$. Substituting in for each y_t^j using equation (36) implies that:

$$\textit{Nominal Total Cost} = Y_t \left(\int_0^1 \left(P_t^j \right)^{1-\theta} dj \right)^{\frac{\theta}{1-\theta}} \int_0^1 \left(P_t^j \right)^{1-\theta} dj = Y_t \left(\int_0^1 \left(P_t^j \right)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}.$$

Since the intermediary produces aggregate output competitively its price, which is equal to nominal marginal cost, is given by:

$$P_t = \left(\int_0^1 \left(P_t^j \right)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \tag{37}$$

This expression for the price level can be substituted into equation (36) to yield a simpler expression for the intermediary's demand for good j:

$$y_t^j = \left(\frac{P_t^j}{P_t}\right)^{-\theta} y_t. \tag{38}$$

The assumption of two-period price staggering implies that equation (37) can be written as:

$$P_t = \left(\int_0^{\frac{1}{2}} \left(X_t^p\right)^{1-\theta} di + \int_{\frac{1}{2}}^{1} \left(X_{t-1}^p\right)^{1-\theta} di\right)^{\frac{1}{1-\theta}} = \left(\frac{1}{2}\left(X_t^p\right)^{1-\theta} + \frac{1}{2}\left(X_{t-1}^p\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

where X_t^p is defined as the price reset in period t. Dividing through by P_t yields:

$$1 = \left(\frac{1}{2}\left(\frac{X_t^p}{P_t}\right)^{1-\theta} + \frac{1}{2}\left(\frac{X_{t-1}^p}{P_{t-1}} \cdot \frac{1}{\Pi_t}\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}. \tag{39}$$

B.3 The Household's Problem

Household i chooses $\{c_t^i, \frac{M_t^i}{P_t}, h_t^i, k_t^i\}_{t=0}^{\infty}$ to maximize its utility (equation (3p)) subject to its budget constraint and the evolution of the capital stock (equations (29p) and (5p)), taking as given the nominal interest rate, the gross inflation rate, the real rental rate on capital, and the real wage rate on labor. Specifically, household i solves:

$$\max_{\left\{c_t^i, \frac{M_t^i}{P_t}, h_t^i, k_t^i\right\}_{t=0}^{\infty}} E_t\left[\sum_{t=0}^{\infty} \ln\left[\left(b\left(c_t^i\right)^v + (1-b)\left(\frac{M_t^i}{P_t}\right)^v\right)^{\frac{1}{v}}\right] - \eta \ln\left[1 - h_t^i\right]\right]$$

subject to:

$$B_t^i + M_t^i \leq R_{t-1}B_{t-1}^i + M_{t-1}^i + P_t w_t h_t^i + P_t r_t k_t^i - P_t c_t^i - P_t k_t^i J^{-1}\left(\frac{k_{t+1}^i}{k_t^i} - 1 + \delta\right).$$

The first-order conditions for real money balances, consumption, and capital supply are identical to those given by equations (10) to (12), and can be rearranged in the same way as they were in the staggered-wage model in order to yield equations (13) to (15). The first-order condition for h_t^i is now:

$$0 = \frac{\beta^t}{c_t^i} \cdot \frac{b\left(c_t^i\right)^v}{b\left(c_t^i\right)^v + (1-b)\left(\frac{M_t^i}{P_t}\right)^v} w_t - \frac{\beta^t \eta}{1 - h_t^i},$$

which simplifies to:

$$U_c\left(c_t, R_t\right) w_t = \frac{\eta}{1-h_t}. \tag{40}$$

Since all households receive the same real wage and rental rate, and hence supply the same amounts of labor and capital, their real wealth and thus $\{c_t^i, \frac{M_t^i}{P_t}, h_t^i, k_t^i\}_{t=0}^{\infty}$ will be identical. As a result, the households' first-order conditions (equations (13) to (15), and (40)) can be written without the i superscripts.

B.4 Solving the Fully Specified Model

Equilibrium is an allocation $\{\{h_t^j\}_{j=0}^1, h_t, \{k_t^j\}_{j=0}^1, k_t, c_t, \frac{M_t}{P_t}, \{y_t^j\}_{j=0}^1, y_t\}_{t=0}^{\infty}$ and a sequence $\{\Pi_t, \frac{X_t^p}{P_t}, w_t, r_t, \mu_t, R_t, mc_t\}_{t=0}^{\infty}$. The equilibrium allocation and sequence satisfy the following conditions: (i) the first-order conditions from the firms' cost-minimization problem (26p) and profit-maximization problem (27p) (equations (31) and (33)); (ii) the first-order conditions from the intermediary's cost-minimization problem (28p) (equations (38) and (39)); (iii) the first-order conditions from the households' utility-maximization problems (30p) (equations (13) to (15) and (40)); (iv) the monetary authority follows (8p); (v) the goods market clears ($y_t = c_t + k_t J^{-1}\left(\frac{k_{t+1}}{k_t} - 1 + \delta\right)$); and (vi) factor markets clear ($h_t = \int_0^1 h_t^j dj$ and $k_t = \int_0^1 k_t^j dj$). (This is given the initial conditions, k_0, μ_{-1}, $\frac{M_{-1}}{P_{-1}}$, $\frac{X_{-1}^p}{P_{-1}}$, and the sequence of monetary policy shocks $\{\varepsilon_t\}_{t=0}^{\infty}$.) The model's log-linearized first-order conditions are given in table B.1. The model is calibrated with the parameter values given in table 1 of the paper. The log-linearized first-order conditions given in table B.1 can be reduced to the system of difference equations described in section 2.7.1 of the paper.

B.5 Solving the Simplified Model

Equilibrium in the core model of section 2.7.2 of the paper is an allocation $\{y_t\}_{t=0}^{\infty}$ and a sequence $\{\frac{X_t^p}{P_t}, R_t\}_{t=0}^{\infty}$ that satisfy equations (10p), (12p), and (31p), with the equilibrium conditions noted in points (a) to (f) of section 2.7.2 imposed. Specifically,

$$\frac{1}{y_t} \cdot \frac{1}{2-\frac{1}{R_t}} = R_t \beta E_t \left[\frac{1}{y_{t+1}} \cdot \frac{1}{2-\frac{1}{R_{t+1}}} \cdot \frac{\left(2-\left(X_{t+1}^p/P_{t+1}\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}}{X_t^p/P_t} \right], \tag{41}$$

Table B.1

$\widehat{\mu}_t = \zeta \widehat{\mu}_{t-1} + \varepsilon_t$	Eq. (8p)
$\widehat{\frac{M_t}{P_t}} = \widehat{c}_t - \left(\frac{1}{1-v}\right)\left(\frac{\frac{1}{R^*}}{1-\frac{1}{R^*}}\right)\widehat{R}_t$	Eq. (13)
$-\rho_{cc} E_t \widehat{c}_{t+1} - \rho_{cr} E_t \widehat{R}_{t+1} = -\rho_{cc}\widehat{c}_t + (1-\rho_{cr})\widehat{R}_t - E_t \widehat{\Pi}_{t+1}$	Eq. (14)
$\frac{1}{1+\delta\beta-\beta}\left(\widehat{R}_t - E_t\widehat{\Pi}_{t+1}\right) = E_t \widehat{r}_{t+1} + \frac{J^{-1''}(\delta)}{1+\delta\beta-\beta}\left(\beta E_t \widehat{k}_{t+2} - (1+\beta)\widehat{k}_{t+1} + \widehat{k}_t\right)$	Eq. (15)
$\widehat{h}_t^j = \widehat{y}_t^j - \alpha \widehat{w}_t + \alpha \widehat{r}_t$	Eq. (31)
$\widehat{k}_t^j = \widehat{y}_t^j + (1-\alpha)\widehat{w}_t - (1-\alpha)\widehat{r}_t$	Eq. (31)
$\widehat{mc}_t = (1-\alpha)\widehat{w}_t + \alpha \widehat{r}_t = 0$	Eq. (31)
$\widehat{\frac{X_t^p}{P_t}} = \left(\frac{1}{1+\beta}\right)\widehat{mc}_t + \left(\frac{\beta}{1+\beta}\right)E_t \widehat{mc}_{t+1} + \left(\frac{\beta}{1+\beta}\right)E_t \widehat{\Pi}_{t+1}$	Eq. (33, 13p)
$\widehat{y}_t^j = \widehat{y}_t - \theta \widehat{\frac{X_t^p}{P_t}}$ for $j \in \left[0, \frac{1}{2}\right]$	Eq. (38)
$\widehat{y}_t^j = \widehat{y}_t - \theta \widehat{\frac{X_{t-1}^p}{P_{t-1}}} + \theta \widehat{\Pi}_t$ for $j \in \left(\frac{1}{2}, 1\right]$	Eq. (38)
$\widehat{\Pi}_t = \widehat{\frac{X_t^p}{P_t}} + \widehat{\frac{X_{t-1}^p}{P_{t-1}}}$	Eq. (39, 32p)
$\widehat{w}_t = \rho_{hh}\widehat{h}_t - \rho_{cc}\widehat{c}_t - \rho_{cr}\widehat{R}_t$	Eq. (40)
$\widehat{h}_t = \frac{1}{2}\widehat{h}_t^k + \frac{1}{2}\widehat{h}_t^l$ where $k \in \left[0, \frac{1}{2}\right]$ and $l \in \left(\frac{1}{2}, 1\right]$	H-Clearing
$\widehat{k}_t = \frac{1}{2}\widehat{k}_t^k + \frac{1}{2}\widehat{k}_t^l$ where $k \in \left[0, \frac{1}{2}\right]$ and $l \in \left(\frac{1}{2}, 1\right]$	K-Clearing
$\widehat{y}_t = c\widehat{c}_t + (1-c)\left(\frac{1}{\delta}E_t \widehat{k}_{t+1} - \frac{1-\delta}{\delta}\widehat{k}_t\right)$	Y-Clearing

$$
\begin{aligned}
\frac{X_t^p}{P_t} &= \left(\frac{\theta}{\theta-1}\right) \qquad (42)\\
&\times \frac{\dfrac{\eta y_t^2\left(2-\frac{1}{R_t}\right)}{1-\frac{y_t}{2}\left[\left(\frac{X_t}{P_t}\right)^{-\theta}+\left(2-\left(\frac{X_t}{P_t}\right)^{1-\theta}\right)^{\frac{-\theta}{1-\theta}}\right]} + E_t\left[\dfrac{\frac{1}{R_t}\eta y_{t+1}^2\left(2-\frac{1}{R_{t+1}}\right)}{1-\frac{y_{t+1}}{2}\left[\left(\frac{X_{t+1}}{P_{t+1}}\right)^{-\theta}+\left(2-\left(\frac{X_{t+1}}{P_{t+1}}\right)^{1-\theta}\right)^{\frac{-\theta}{1-\theta}}\right]}\dfrac{\left(\frac{X_t}{P_t}\right)^{1+\theta}}{\left(2-\left(\frac{X_{t+1}}{P_{t+1}}\right)^{1-\theta}\right)^{\frac{1+\theta}{1-\theta}}}\right]}{y_t + E_t\left[\frac{1}{R_t}y_{t+1}\dfrac{\left(\frac{X_t}{P_t}\right)^{\theta}}{\left(2-\left(\frac{X_{t+1}}{P_{t+1}}\right)^{1-\theta}\right)^{\frac{\theta}{1-\theta}}}\right]},
\end{aligned}
$$

$$
\text{and} \quad \frac{\left(2-\left(X_t^p/P_t\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}}{X_{t-1}^p/P_{t-1}} y_t = y_{t-1}\mu^* \exp\left[\varepsilon_t\right]. \qquad (43)
$$

This is given y_{-1} and $\frac{X_{-1}^p}{P_{-1}}$ and the sequence of monetary policy shocks $\{\varepsilon_t\}_{t=0}^{\infty}$.

Equations (41) to (43) can be log-linearized to yield equations (33p) to (35p) in section 2.7.2 of the paper. Of the three equations that characterize equilibrium in the simplified staggered-price model, only equation (19) is somewhat arduous to log-linearize. This equation log-linearizes as follows:

$$\begin{aligned}\widehat{X}_t^p - \widehat{P}_t &= \frac{1}{1+\frac{1}{R^*}}\left((2+\rho_{hh})\,\widehat{y}_t + \frac{\frac{1}{R^*}}{2-\frac{1}{R^*}}\widehat{R}_t\right) - \frac{1}{1+\frac{1}{R^*}}\widehat{y}_t \\ &+ \frac{\frac{1}{R^*}}{1+\frac{1}{R^*}}\left((2+\rho_{hh})E_t\widehat{y}_{t+1} + \frac{\frac{1}{R^*}}{2-\frac{1}{R^*}}E_t\widehat{R}_{t+1} + (1+\theta)\left(\widehat{X}_t^p - \widehat{P}_t + E_t\widehat{X}_{t+1}^p - E_t\widehat{P}_{t+1}\right) - \widehat{R}_t\right) \\ &- \frac{\frac{1}{R^*}}{1+\frac{1}{R^*}}\left(E_t\widehat{y}_{t+1} + \theta\left(\widehat{X}_t^p - \widehat{P}_t + E_t\widehat{X}_{t+1}^p - E_t\widehat{P}_{t+1}\right) - \widehat{R}_t\right)\end{aligned}$$

where ρ_{hh}, the elasticity of labor substitution, is: $\frac{V''(h^*)h^*}{V'(h^*)} = \frac{h^*}{1-h^*}$. This rearranges to:

$$\widehat{X}_t^p - \widehat{P}_t = (1+\rho_{hh})\left(\widehat{y}_t + \frac{1}{R^*}E_t\widehat{y}_{t+1}\right) + \frac{\frac{1}{R^*}}{2-\frac{1}{R^*}}\left(\widehat{R}_t + \frac{1}{R^*}E_t\widehat{R}_{t+1}\right) + \frac{1}{R^*}\left(E_t\widehat{X}_{t+1}^p - E_t\widehat{P}_{t+1}\right).$$

Setting β, the discount factor (and, by implication, R^*, the gross nominal interest rate) equal to unity yields equation (35p) with γ equal to $(1+\rho_{hh})$ and ϕ equal to 1.

The equilibrium paths of $\widehat{y}_t$, $\widehat{\frac{X_t^p}{P_t}}$, and $\widehat{R}_t$ (which are given by equation (36p)) can be found from the log-linearized equations (33p) to (35p) by following exactly the same steps outlined in section 1.7.2 of the paper and presented in more detail in section A.6 of the appendix.

C Derivation of the Staggered Price Model with Firm-Specific Factors

C.1 The Firm's Problem

As noted in section 3.2 of the paper the problem for firms in the staggered price model with firm-specific labor inputs is very similar to the problem faced by firms in the staggered price model with homogeneous labor; the differences are that h_t^j and k_t^j now have the interpretation of being firm j's demand for its specific labor and capital inputs and that firms now face real wage and real rental rates (w_t^j and r_t^j) associated with their specific factors. The problem that firms first solve is therefore:

$$\min_{\{h_t^j, k_t^j\}} w_t^j h_t^j + r_t^j k_t^j \text{ subject to } \left(h_t^j\right)^{1-\alpha}\left(k_t^j\right)^{\alpha} \geq y_t^j.$$

The solutions that emerge from the problem are:

$$h_t^j = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} y_t^j \left(\frac{w_t^j}{r_t^j}\right)^{-\alpha},\ k_t^j = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} Y_t^j \left(\frac{w_t^j}{r_t^j}\right)^{1-\alpha},\ \text{and } mc_t^j = \left(\frac{w_t^j}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t^j}{\alpha}\right)^{\alpha}. \tag{44}$$

Firm j's price-setting problem is identical to that solved in (32) of section B.1 and so the solution is very similar to that given by equation (33). Note, however, that since the real wage and rental rate differ across firms, real marginal cost will also differ; consequently mc_t^j is written *with* its j superscript. The first-order condition for prices, therefore, for a firm j which resets its price in periods $\{2k\}_{k=0}^{\infty}$ is:

$$\frac{X_t^p}{P_t} = \left(\frac{\theta}{\theta-1}\right) \frac{y_t mc_t^j + E_t\left[\left(\frac{\Pi_{t+1}}{R_t}\right) y_{t+1} mc_{t+1}^j \left(\frac{1}{\Pi_{t+1}}\right)^{-\theta}\right]}{y_t + \left(\frac{1}{R_t}\right) E_t\left[y_{t+1}\left(\frac{1}{\Pi_{t+1}}\right)^{-\theta}\right]}. \tag{45}$$

This is equation (40p) in section 3.7.2 of the paper.

C.2 The Intermediary's Problem

The intermediary's problem is identical to that solved in section B.2.

C.3 The Household's Problem

The household's problem, that of choosing $\{c_t^i, \frac{M_t^i}{P_t}, h_t^i, k_t^i\}_{t=0}^{\infty}$, changes only to reflect the fact that with firm-specific factors real wages (w_t^i) and real rents (r_t^i) as well as hours worked (h_t^i) and capital supplied (k_t^i) vary across households. The household's problem becomes:

$$\max_{\left\{c_t^i, \frac{M_t^i}{P_t}, h_t^i, k_t^i\right\}_{t=0}^{\infty}} E_t\left[\sum_{t=0}^{\infty} \ln\left[\left(b\left(c_t^i\right)^v + (1-b)\left(\frac{M_t^i}{P_t}\right)^v\right)^{\frac{1}{v}}\right] - \eta \ln\left[1-h_t^i\right]\right] \tag{46}$$

subject to:

$$B_t^i + M_t^i \leq R_{t-1}B_{t-1}^i + M_{t-1}^i + P_t w_t^i h_t^i + P_t r_t^i k_t^i - P_t c_t^i - P_t k_t^i J^{-1}\left(\frac{k_{t+1}^i}{k_t^i} - 1 + \delta\right),$$

taking as given the nominal interest rate, the gross inflation rate, the real rental rate on its capital, and the real wage rate on its labor. The first-order conditions for real money balances and consumption are identical to those given by equation (10) and (11) and can be rearranged in the same way that they were in the two previous models to yield equations (13) and (14). The first-order conditions for k_t^i and h_t^i are now:

$$\begin{aligned}
&\frac{\beta^t}{c_t^i}\frac{b\left(c_t^i\right)^v}{b\left(c_t^i\right)^v+(1-b)\left(\frac{M_t^i}{P_t}\right)^v}\left(J^{-1'}\left(\frac{k_{t+1}^i}{k_t^i}-1+\delta\right)\right)\\
=\; &E_t\left[\frac{\beta^{t+1}}{c_{t+1}^i}\frac{b\left(c_{t+1}^i\right)^v}{b\left(c_{t+1}^i\right)^v+(1-b)\left(\frac{M_{t+1}^i}{P_{t+1}}\right)^v}\left(r_{t+1}^i-J^{-1}\left(\frac{k_{t+2}^i}{k_{t+1}^i}-1+\delta\right)-\frac{k_{t+2}^i}{k_{t+1}^i}J^{-1'}\left(\frac{k_{t+2}^i}{k_{t+1}^i}-1+\delta\right)\right)\right]
\end{aligned}$$

$$\text{and } 0=\frac{\beta^t}{c_t^i}\cdot\frac{b\left(c_t^i\right)^v}{b\left(c_t^i\right)^v+(1-b)\left(\frac{M_t^i}{P_t^i}\right)^v}\cdot w_t^i-\frac{\beta^t\eta}{1-h_t^i},$$

which simplify to:

$$\begin{aligned}
&U_c\left(c_t,R_t\right)\left(J^{-1'}\left(\frac{k_{t+1}^i}{k_t^i}-1+\delta\right)\right) &\quad (47)\\
=\; &E_t\left[\beta U_c\left(c_{t+1},R_{t+1}\right)\left(r_{t+1}^i-J^{-1}\left(\frac{k_{t+2}^i}{k_{t+1}^i}-1+\delta\right)-\frac{k_{t+2}^i}{k_{t+1}^i}J^{-1'}\left(\frac{k_{t+2}^i}{k_{t+1}^i}-1+\delta\right)\right)\right]
\end{aligned}$$

$$\text{and } U_c\left(c_t,R_t\right)w_t^i=\frac{\eta}{1-h_t^i}. \quad (48)$$

I make the same assumption as in the staggered-wage model that asset portfolios can be constructed so as to provide the household with complete insurance against any idiosyncratic risk. Consequently, a household's wealth is independent of the wage and rental rate that it faces and the amount of labor and capital that it supplies. This allows me to write the households' first-order conditions (equations (13), (14), (47), and (48)) without the i subscripts on consumption or real money balances.

C.4 Solving the Fully Specified Model

Equilibrium is an allocation $\{\{h_t^j\}_{j=0}^1,\{h_t^i\}_{i=0}^1,\{k_t^j\}_{j=0}^1,\{k_t^i\}_{i=0}^1,c_t,\frac{M_t}{P_t},\{y_t^j\}_{j=0}^1,y_t\}_{t=0}^\infty$ and a sequence $\{\Pi_t,\frac{X_t^p}{P_t},\{w_t^j\}_{j=0}^1,\{w_t^i\}_{i=0}^1,\{r_t^j\}_{j=0}^1,\{r_t^i\}_{i=0}^1,\mu_t,R_t,\{mc_t^j\}_{j=0}^1\}_{t=0}^\infty$. The equilibrium allocation and sequence satisfy the following conditions: (i) the first-order conditions from the firms' cost-minimization problem (37p) and profit-maximization problem (27p) (equations (44) and (45)) ; (ii) the first-order conditions from the intermediary's cost minimization problem (28p) (equations (38) and (39)); (iii) the first-order conditions from the households' utility-maximization problem (39p) (equations (13), (14), (47), and (48)); (iv) the monetary authority follows (8p); (v) the goods market clears ($y_t = c_t + \int_0^1 k_t^i J^{-1}\left(\frac{k_{t+1}^i}{k_t^i} - 1 + \delta\right) di$); and (vi) factor markets clear ($h_t^j = h_t^i$, $k_t^j = k_t^i$, $w_t^j = w_t^i$, and $r_t^j = r_t^i$). (This is given the initial conditions, k_0, μ_{-1}, $\frac{M_{-1}}{P_{-1}}$, $\frac{X_{-1}^p}{P_{-1}}$, and the sequence of monetary policy shocks $\{\varepsilon_t\}_{t=0}^{\infty}$.) The model's log-linearized first-order conditions are given in table C.1.

Table C.1

$\widehat{\mu}_t = \zeta\widehat{\mu}_{t-1} + \varepsilon_t$	Eq. (8p)
$\widehat{\frac{M_t}{P_t}} = \widehat{c}_t - \left(\frac{1}{1-v}\right)\left(\frac{\frac{1}{R^*}}{1-\frac{1}{R^*}}\right)\widehat{R}_t$	Eq. (13)
$-\rho_{cc}E_t\widehat{c}_{t+1} - \rho_{cr}E_t\widehat{R}_{t+1} = -\rho_{cc}\widehat{c}_t + (1-\rho_{cr})\,\widehat{R}_t - E_t\widehat{\Pi}_{t+1}$	Eq. (14)
$\widehat{y}_t^j = y - \theta\widehat{\frac{X_t^p}{P_t}}$ for $j \in \left[0, \frac{1}{2}\right]$	Eq. (38)
$\widehat{y}_t^j = \widehat{y}_t - \theta\widehat{\frac{X_{t-1}^p}{P_{t-1}}} + \theta\widehat{\Pi}_t$ for $j \in \left(\frac{1}{2}, 1\right]$	Eq. (38)
$\widehat{\Pi}_t = \widehat{\frac{X_t^p}{P_t}} + \widehat{\frac{X_{t-1}^p}{P_{t-1}}}$	Eq. (39, 32p)
$\widehat{h}_t^j = \widehat{y}_t^j - \alpha\widehat{w}_t^j + \alpha\widehat{r}_t^j$	Eq. (44)
$\widehat{k}_t^j = \widehat{y}_t^j + (1-\alpha)\,\widehat{w}_t^j - (1-\alpha)\,\widehat{r}_t^j$	Eq. (44)
$\widehat{mc}_t^j = (1-\alpha)\,\widehat{w}_t^j + \alpha\widehat{r}_t^j = 0$	Eq. (44)
$\widehat{\frac{X_t^p}{P_t}} = \left(\frac{1}{1+\beta}\right)\widehat{mc}_t^j + \left(\frac{\beta}{1+\beta}\right)E_t\widehat{mc}_{t+1}^j + \left(\frac{\beta}{1+\beta}\right)E_t\widehat{\Pi}_{t+1}$ for $j \in \left[0, \frac{1}{2}\right]$	Eq. (45, 40p)
$\frac{1}{1+\delta\beta-\beta}\left(\widehat{R}_t - E_t\widehat{\Pi}_{t+1}\right) = E_t\widehat{r}_{t+1}^i + \frac{J^{-1''}(\delta)}{1+\delta\beta-\beta}\left(\beta E_t\widehat{k}_{t+2}^i - (1+\beta)\,\widehat{k}_{t+1}^i + \widehat{k}_t^i\right)$	Eq. (47)
$\widehat{w}_t^i = \rho_{hh}\widehat{h}_t^i - \rho_{cc}\widehat{c}_t - \rho_{cr}\widehat{R}_t$	Eq. (48)
$\widehat{h}_t^j = \widehat{h}_t^i$ and $\widehat{w}_t^j = \widehat{w}_t^i$where $\forall\ i = j \in [0,1]$	H-Clearing
$\widehat{k}_t^j = \widehat{k}_t^i$ and $\widehat{r}_t^j = \widehat{r}_t^i$where $\forall\ i = j \in [0,1]$	K-Clearing
$\widehat{y}_t = c\widehat{c}_t + (1-c)\left(\frac{1}{\delta}\int_0^1 E_t\widehat{k}_{t+1}^i di - \frac{1-\delta}{\delta}\int_0^1 \widehat{k}_t^i di\right)$	Y-Clearing

The model is calibrated with the parameter values given in table 1 of the paper. The log-linearized first-order conditions given in table C.1 can be reduced to the system of difference equations described in section 3.7.1 of the paper.

C.5 Solving the Simplified Model

Equilibrium in the core model of section 3.7.2 of the paper is an allocation $\{y_t\}_{t=0}^{\infty}$ and a sequence of prices $\{\frac{X_t^p}{P_t}, R_t\}_{t=0}^{\infty}$ that satisfy equations (10p), (12p), and (40p), with the equilibrium conditions noted in points (a) to (f) of section 3.7.2 imposed. Specifically, equilibrium is characterized by equations (41) and (43) from section B.6, as well as

$$\frac{X_t^p}{P_t}=\left(\frac{\theta}{\theta-1}\right)\frac{\frac{\eta y_t^2\left(2-\frac{1}{R_t}\right)}{1-y_t\left(X_t^p/P_t\right)^{-\theta}}+E_t\left[\frac{\frac{1}{R_t}\eta y_{t+1}^2\left(2-\frac{1}{R_{t+1}}\right)}{1-y_{t+1}\left(2-\left(X_{t+1}^p/P_{t+1}\right)^{1-\theta}\right)^{\frac{-\theta}{1-\theta}}}\frac{\left(X_t^p/P_t\right)^{1+\theta}}{\left(2-\left(X_{t+1}^p/P_{t+1}\right)^{1-\theta}\right)^{\frac{1+\theta}{1-\theta}}}\right]}{y_t+E_t\left[\frac{1}{R_t}y_{t+1}\frac{\left(X_t^p/P_t\right)^{\theta}}{\left(2-\left(X_{t+1}^p/P_{t+1}\right)^{1-\theta}\right)^{\frac{\theta}{1-\theta}}}\right]}. \tag{49}$$

This is given y_{-1} and $\frac{X_{-1}^p}{P_{-1}}$ and the sequence of monetary policy shocks $\{\varepsilon_t\}_{t=0}^{\infty}$.

Equations (41), (43), and (49) can be log-linearized to equations (33p) to (35p), that are then used in section 3.7.2 of the paper to find the equilibrium paths of $\widehat{y}_t$, $\widehat{\frac{X_t^p}{P_t}}$, and $\widehat{R}_t$. Of the three equations that characterize equilibrium only equation (49) is somewhat difficult to log-linearize. This equation log-linearizes as follows:

$$\begin{aligned}\widehat{X}_t^p-\widehat{P}_t \quad = \quad & \frac{1}{1+\frac{1}{R^*}}\left(\left(2+\rho_{hh}\right)\widehat{y}_t+\frac{\frac{1}{R^*}}{2-\frac{1}{R^*}}\widehat{R}_t\right)-\frac{\theta\rho_{hh}}{1+\frac{1}{R^*}}\left(\widehat{X}_t^p-\widehat{P}_t\right)-\frac{1}{1+\frac{1}{R^*}}\widehat{y}_t \\ & +\frac{\frac{1}{R^*}}{1+\frac{1}{R^*}}\left((2+\rho_{hh})E_t\widehat{y}_{t+1}+\frac{\frac{1}{R^*}}{2-\frac{1}{R^*}}E_t\widehat{R}_{t+1}+(1+\theta)\left(\widehat{X}_t^p-\widehat{P}_t+E_t\widehat{X}_{t+1}^p-E_t\widehat{P}_{t+1}\right)-\widehat{R}_t\right) \\ & +\frac{\frac{\theta\rho_{hh}}{R^*}}{1+\frac{1}{R^*}}\left(E_t\widehat{X}_{t+1}^p-E_t\widehat{P}_{t+1}\right)-\frac{\frac{1}{R^*}}{1+\frac{1}{R^*}}\left(E_t\widehat{y}_{t+1}+\theta\left(\widehat{X}_t^p-\widehat{P}_t+E_t\widehat{X}_{t+1}^p-E_t\widehat{P}_{t+1}\right)-\widehat{R}_t\right).\end{aligned}$$

This rearranges to

$$\begin{aligned}\left(1+\theta\rho_{hh}\right)\left(\widehat{X}_t^p-\widehat{P}_t\right) \quad = \quad & \left(1+\rho_{hh}\right)\left(\widehat{y}_t+\frac{1}{R^*}E_t\widehat{y}_{t+1}\right)+\frac{\frac{1}{R^*}}{2-\frac{1}{R^*}}\left(\widehat{R}_t+\frac{1}{R^*}E_t\widehat{R}_{t+1}\right) \\ & +\frac{1+\theta\rho_{hh}}{R^*}\left(E_t\widehat{X}_{t+1}^p-E_t\widehat{P}_{t+1}\right).\end{aligned}$$

Dividing through on both sides by $(1+\theta\rho_{hh})$ and setting β, the discount factor (and by implication, R^*, the gross nominal interest rate) equal to unity yields equation (35p) with γ equal to $\frac{1+\rho_{hh}}{1+\theta\rho_{hh}}$ and ϕ equal to $\frac{1}{1+\theta\rho_{hh}}$.

The equilibrium paths of $\widehat{y}_t$, $\widehat{\frac{X_t^p}{P_t}}$, and $\widehat{R}_t$ (which are given by equation (41p)) can be found from the log-linearized system defined by equations (33p) to (35p); the steps involved are exactly the same as those outlined in section 1.7.2 of the paper and presented in more detail in section A.6.

D Comparison with Chari, Kehoe, and McGratten's Firm-Specific Factor Results

Chari, Kehoe, and McGratten (2000) incorporate firm-specific factors into their model by assuming that firms produce their differentiated outputs using homogeneous labor and some fixed factor specific to their production process. Specifically, Chari *et al.* assume that firms face a production function of:

$$y_t^j = \left(h_t^j\right)^{1-\psi} j^{\psi}$$

where h_t^j is firm j's use of the homogeneous labor input, j is firm j's fixed, undepreciable, firm-specific input, and ψ represents the elasticity of output with respect to the firm-specific input. (Clearly, assuming $\psi = 0$ returns the model to that of section 2 of the paper.) It is assumed that all firms are endowed with identical quantities of their firm-specific input and that each firm's specific input is useful only to itself. Consequently, firms face no price for their firm-specific input. The firm's demand for homogeneous labor can be found by simply rearranging the production function:

$$h_t^j = \left(y_t^j\right)^{\frac{1}{1-\psi}} (j)^{-\frac{\psi}{1-\psi}} . \tag{50}$$

Since the wage bill ($w_t h_t^j$) is the only cost of production, total and marginal cost are given by:

$$\begin{aligned} tc_t^j &= w_t \left(y_t^j\right)^{\frac{1}{1-\psi}} (j)^{-\frac{\psi}{1-\psi}} \text{ and} \\ mc_t^j &= \left(\frac{1}{1-\psi}\right) w_t \left(y_t^j\right)^{\frac{\psi}{1-\psi}} (j)^{-\frac{\psi}{1-\psi}} . \end{aligned} \tag{51}$$

The firm's price-setting problem is identical to that solved in section 2.2 (section B.2) and section 3.2 (section C.2). The first-order conditions for prices for firms who reset their prices in periods $\{2k\}_{k=0}^{\infty}$ (assuming two-period price stickiness) is given by equation (31p) in section 2.2 of the paper (with the superscript js on marginal cost retained). The log-linearized version of equation (31p) is similar to that given in tables B.1 and C.1; since I will be obtaining analytical solutions, however, I employ the simplifying approximation that $\beta = 1$ (and implicitly $R^* = 1$). This yields:

$$\widehat{\frac{X_t^p}{P_t}} = \frac{1}{2}\widehat{mc}_t^j + \frac{1}{2}E_t\widehat{mc}_{t+1}^j + \frac{1}{2}E_t\widehat{\Pi}_{t+1} \text{ for } j \in \left[0, \frac{1}{2}\right] \tag{52}$$

where $\widehat{mc}_t^j$ and $E_t\widehat{mc}_{t+1}^j$ are given by the log-linear approximations of equation (51):

$$\widehat{mc}_t^j = \widehat{w}_t + \frac{\psi}{1-\psi}\widehat{y}_t^j \text{ and } E_t\widehat{mc}_{t+1}^j = E_t\widehat{w}_{t+1} + \frac{\psi}{1-\psi}E_t\widehat{y}_{t+1}^j. \tag{53}$$

The intermediary's problem is identical to that outlined in section 2.3 of the paper; its demand for the jth good is given by equation (38) while the price index for the aggregate good is given by equation (32) in section 2.2 of the paper (or equation (39) in section B.2). The household's problem is similar to that outlined in section 2.4 of the paper, although without capital. The simplifying assumption that $v \to -\infty$ implies that the household's money demand curve, Euler equation, and labor supply schedule are those given by $\frac{M_t}{P_t} = y_t$, equation (10p), and $w_t = V'(h_t)\, c_t \left(2 - \frac{1}{R_t}\right)$. The simplifying assumption that $\zeta = 0$ allows the money growth process to be written as equation (12p).

Equation (53) can be simplified by substituting out for $\widehat{y}_t^j$ and $E_t\widehat{y}_{t+1}^j$ (from the intermediary's demand for the differentiated goods) and for $\widehat{w}_t$ and $E_t\widehat{w}_{t+1}$ (from the household's labor supply curve). The log-linearized approximation of the intermediary's demand for the differentiated goods, given in tables B.1 and C.1, implies that for firms who reset their prices in period t, $\widehat{y}_t^j$ and $E_t\widehat{y}_{t+1}^j$ are:

$$\widehat{y}_t^j = \widehat{y}_t - \theta\widehat{\frac{X_t^p}{P_t}} \text{ and } E_t\widehat{y}_{t+1}^j = E_t\widehat{y}_{t+1} - \theta\widehat{\frac{X_t^p}{P_t}} + \theta E_t\widehat{\Pi}_{t+1}. \tag{54}$$

The log-linearized approximation of the households' labor supply curve (with $R^* = 1$) implies that $\widehat{w}_t$ and $E_t\widehat{w}_{t+1}$ are given by:

$$\widehat{w}_t = \rho_{hh}\widehat{h}_t + \widehat{c}_t + \widehat{R}_t \text{ and } E_t\widehat{w}_{t+1} = \rho_{hh}E_t\widehat{h}_{t+1} + E_t\widehat{c}_{t+1} + E_t\widehat{R}_{t+1}. \tag{55}$$

Combining equations (53), (54), and (55) implies that:

$$\widehat{mc}_t^j = \rho_{hh}\widehat{h}_t + \widehat{c}_t + \widehat{R}_t + \frac{\psi}{1-\psi}\left(\widehat{y}_t - \theta\widehat{\frac{X_t^p}{P_t}}\right) \text{ and} \tag{56}$$

$$E_t\widehat{mc}_{t+1}^j = \rho_{hh}E_t\widehat{h}_{t+1} + E_t\widehat{c}_{t+1} + E_t\widehat{R}_{t+1} + \frac{\psi}{1-\psi}\left(E_t\widehat{y}_{t+1} - \theta\widehat{\frac{X_t^p}{P_t}} + \theta E_t\widehat{\Pi}_{t+1}\right). \tag{57}$$

Equations (56) and (57) can be simplified further.[1] First, I can substitute for $E_t\widehat{\Pi}_{t+1} - \widehat{\frac{X_t^p}{P_t}}$

[1]Equations (56) and (57) highlight the point (emphasized in section 4 of the paper) that including firm-specific factors in the model creates a feedback effect between price adjustment and marginal cost. It can be seen from the last term in each equation that an increase in the price set by the firm for its differentiated output reduces the demand for its output and in turn reduces its marginal cost. What is also clear from equations (56) and (57) is that removing firm-specific factors from the model (by setting ψ equal to zero) eliminates the last term from each equation and thus removes this crucial feedback effect from the model.

with $E_t[\widehat{\frac{X^p_{t+1}}{P_{t+1}}}]$ by noting that the price index for the aggregate good (equation (32p) or (39)) can be written as $\Pi_t = \left(\frac{\left(X^p_{t-1}/P_{t-1}\right)^{1-\theta}}{2-\left(X^p_t/P_t\right)^{1-\theta}}\right)^{\frac{1}{1-\theta}}$. This can then be log-linearized, brought forward one period, and expressed in terms of period t expectations to yield $E_t\widehat{\Pi}_{t+1} = E_t[\widehat{\frac{X^p_{t+1}}{P_{t+1}}}] + \widehat{\frac{X^p_t}{P_t}}$. Second, goods-market clearing (in the simplified model with no investment) is $y_t = c_t$, which implies that $\widehat{c}_t = \widehat{y}_t$ and $E_t\widehat{c}_{t+1} = E_t\widehat{y}_{t+1}$. Third, labor-market clearing states that $h_t = \int_0^1 h^j_t dj$. Substituting in the labor demand curve (50) for h^j_t implies that labor-market clearing can be written as:

$$h_t = \int_0^1 \left(y^j_t\right)^{\frac{1}{1-\psi}} (j)^{-\frac{\psi}{1-\psi}}\, dj.$$

Since all firms are endowed with the same amount of the specific factor, j is a constant and can be taken outside the integral so that:

$$h_t = (j)^{-\frac{\psi}{1-\psi}} \int_0^1 \left(y^j_t\right)^{\frac{1}{1-\psi}}\, dj.$$

Substituting in the individual differentiated-good demand curves gives:

$$\begin{aligned} h_t &= (j)^{-\frac{\psi}{1-\psi}} \left(\frac{1}{2} y_t \left(\frac{X^p_t}{P_t}\right)^{-\theta} + \frac{1}{2} Y_t \left(\frac{X^p_{t-1}}{P_{t-1}} \cdot \frac{1}{\Pi_t}\right)^{-\theta}\right)^{\frac{1}{1-\psi}} \\ &= (j)^{-\frac{\psi}{1-\psi}} \left(\frac{1}{2}\right)^{\frac{1}{1-\psi}} (y_t)^{\frac{1}{1-\psi}} \left(\left(\frac{X^p_t}{P_t}\right)^{-\theta} + \left(\frac{X^p_{t-1}}{P_{t-1}} \cdot \frac{1}{\Pi_t}\right)^{-\theta}\right)^{\frac{1}{1-\psi}}. \end{aligned}$$

The log-linearized approximation to this expression is:

$$\widehat{h}_t = \frac{1}{1-\psi}\widehat{y}_t - \frac{\theta}{1-\psi} \underbrace{\left(\widehat{\frac{X^p_t}{P_t}} + \widehat{\frac{X^p_{t-1}}{P_{t-1}}} - \widehat{\Pi}_t\right)}_{=0}.$$

I can thus substitute $\widehat{c}_t = \widehat{y}_t$, $E_t\widehat{c}_{t+1} = E_t\widehat{y}_{t+1}$, $\widehat{h}_t = \frac{1}{1-\psi}\widehat{y}_t$, and $E_t\widehat{h}_{t+1} = \frac{1}{1-\psi}E_t\widehat{y}_{t+1}$ into equations (56) and (57) to yield:

$$\widehat{mc}^j_t = \frac{\rho_{hh}}{1-\psi}\widehat{y}_t + \widehat{y}_t + \widehat{R}_t + \frac{\psi}{1-\psi}\left(\widehat{y}_t - \theta\widehat{\frac{X^p_t}{P_t}}\right) \quad (58)$$

$$E_t\widehat{mc}^j_{t+1} = \frac{\rho_{hh}}{1-\psi}E_t\widehat{y}_{t+1} + E_t\widehat{y}_{t+1} + E_t\widehat{R}_{t+1} + \frac{\psi}{1-\psi}\left(E_t\widehat{y}_{t+1} + \theta E_t\left[\widehat{\frac{X^p_{t+1}}{P_{t+1}}}\right]\right). \quad (59)$$

Equations (58) and (59), along with $E_t\widehat{\Pi}_{t+1} = E_t[\widehat{\frac{X^p_{t+1}}{P_{t+1}}}] + \widehat{\frac{X^p_t}{P_t}}$, can then be substituted into equation (52) so that:

$$\begin{aligned}
\widehat{\frac{X^p_t}{P_t}} &= \frac{1}{2}\left(\left(1+\frac{\rho_{hh}+\psi}{1-\psi}\right)\widehat{y}_t + \widehat{R}_t - \frac{\theta\psi}{1-\psi}\widehat{\frac{X^p_t}{P_t}}\right) \\
&\quad + \frac{1}{2}\left(\left(1+\frac{\rho_{hh}+\psi}{1-\psi}\right)E_t\widehat{y}_{t+1} + E_t\widehat{R}_{t+1} + \frac{\theta\psi}{1-\psi}E_t\left[\widehat{\frac{X^p_{t+1}}{P_{t+1}}}\right]\right) + \frac{1}{2}\left(E_t\left[\widehat{\frac{X^p_{t+1}}{P_{t+1}}}\right] + \widehat{\frac{X^p_t}{P_t}}\right),
\end{aligned}$$

which when rearranged yields:

$$\begin{aligned}
\left(1+\frac{\theta\psi}{1-\psi}\right)\widehat{\frac{X^p_t}{P_t}} &= \left(\left(1+\frac{\rho_{hh}+\psi}{1-\psi}\right)\widehat{y}_t + \widehat{R}_t\right) \\
&\quad + \left(\left(1+\frac{\rho_{hh}+\psi}{1-\psi}\right)E_t\widehat{y}_{t+1} + E_t\widehat{R}_{t+1}\right) + \left(1+\frac{\theta\psi}{1-\psi}\right)E_t\left[\widehat{\frac{X^p_{t+1}}{P_{t+1}}}\right],
\end{aligned}$$

or alternatively:

$$\widehat{\frac{X^p_t}{P_t}} = \gamma(\widehat{y}_t + E_t\widehat{y}_{t+1}) + \phi\left(\widehat{R}_t + E_t\widehat{R}_{t+1}\right) + E_t\left[\widehat{\frac{X^p_{t+1}}{P_{t+1}}}\right] \quad \text{where } \gamma = \frac{1+\frac{\rho_{hh}+\psi}{1-\psi}}{1+\frac{\theta\psi}{1-\psi}} \text{ and } \phi = \frac{1}{1+\frac{\theta\psi}{1-\psi}}. \tag{60}$$

Equation (60) is nearly identical to equation (34p) in section 2.7.2 of the paper, with the only difference being the values of the parameters γ and ϕ. Since the Euler equation and the money growth rule are all still given by equations (10p) and (12p), and since conditions (a) to (e) of section 2.7.2 still hold in this model, their log-linearized approximations are unchanged from those given by equations (33p) and (35p). As in sections 2.7.2 and 3.7.2 of the paper, therefore, the solution to this model is given by:

$$\left\{ \widehat{y}_t,\ \widehat{\frac{X^p_t}{P_t}},\ \widehat{R}_t \right\}_{t=0}^{\infty} = \left\{ \left(\frac{1-\sqrt{\gamma}}{1+\sqrt{\gamma}}\right)^t\left(\frac{1}{1+\sqrt{\gamma}}\right)\cdot\varepsilon_0,\ \left(\frac{1-\sqrt{\gamma}}{1+\sqrt{\gamma}}\right)^t\left(\frac{\sqrt{\gamma}}{1+\sqrt{\gamma}}\right)\cdot\varepsilon_0,\ 0 \right\}_{t=0}^{\infty} \tag{61}$$

where $\gamma = \frac{1+\frac{\rho_{hh}+\psi}{1-\psi}}{1+\frac{\theta\psi}{1-\psi}}$. As in the previous models, monotone damped responses require that the parameter γ is less than one, which occurs when $\theta - \frac{\rho_{hh}}{\psi} > 1$, a condition that is easily satisfied for reasonable parameter values.[2] Thus, Chari, Kehoe, and McGratten's approach to modeling firm-specific factors yields monotone-damped real responses to monetary shocks.

[2] For example, for $5 \leq \theta \leq 20$ and $\rho_{hh} = \frac{H^*}{1-H^*} = \frac{1}{2}$, the elasticity of output with respect to capital, ψ, need only exceed 0.13 for $\theta - \frac{\rho_{hh}}{\psi}$ to be greater than 1.

References

[1] Chari, V. V., Patrick J. Kehoe, and Ellen R. McGratten, "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?" *Econometrica*, 2000, 68, 1151-79.

[2] Huang, Kevin X. D., and Zheng Liu, "Staggered Contracts and Business Cycle Persistence," Federal Reserve Bank of Minneapolis Discussion Paper 127, 1999.